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Dissipative dynamics of coupled quantum dots under quantum measurement

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Abstract

We have studied the dissipative dynamics of a solid-state qubit with an extra electron confined to either one of two coupled quantum dots. Previous theoretical work based on Bloch-type rate equations gave an unphysical uniform occupation probability of the electron in the quantum dots even for non-identical dots. We show that this is due to neglecting higher-order interactions in the analysis. By including higher-order terms, we obtain expected asymmetric occupation probabilities for non-identical dots. Our work demonstrates that the high-order interaction terms can lead to important qualitative impacts on the operation of the qubit.

1. Introduction

Quantum measurement has attracted much attention because of its fundamental importance as well as its relevance to quantum computing. However, there are still vastly different points of view on quantum measurement. On the one hand, the Zeno effect predicts that continuous measurement can freeze the state of a quantum system [1]. On the other hand, interaction with the outside environment such as a detector is expected to lead to the collapse of the state of the measured system. Therefore, reconciling these views by detailing the real mechanisms for selected model systems is very important.

Many experiments on two-level quantum systems (see, e.g., [2–4]) have been performed to study this problem. Together with experimental investigations, some theoretical studies were also devoted to this important topic. Gurvitz *et al* [5, 6] have theoretically studied a mesoscopic qubit-plus-detector system. The qubit consists of two coupled quantum dots (CQDs) while the detector is a quantum point contact (QPC) interacting locally with one of the dots. The authors derived a set of Bloch-type rate equations to describe the measurement effects. Goan *et al* [7] considered the same setup and developed a Lindblad quantum master equation using the

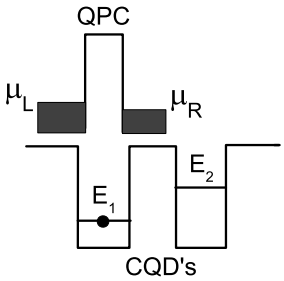

Figure 1. Two coupled quantum dots (CQDs) connected with a quantum point contact (QPC). Initially, all states up to the Fermi energy level ($\mu_{\rm L}$ or $\mu_{\rm R}$) in each reservoir are filled by electrons. E_1 and E_2 denote the energy levels of an extra electron in the left and right quantum dots, respectively. The location of this extra electron will affect the current through the QPC. The quantum state of the CQDs can hence be read out by a current measurement.

quantum trajectory approach for dissipative processes. However, while a reasonable result was obtained for a pair of identical quantum dots in both [5] and [7], these works predicted unreasonable equal occupation probabilities in the dots when their energy levels are different. Subsequently, Li *et al* [8] developed a unified quantum master equation with the Markovian approximation and obtained distinct occupation probabilities for non-identical quantum dots as expected.

In this work, we further study this qubit-plus-detector system following the original approach of Gurvitz *et al* [5]. We have found that the unphysical result of identical occupation probabilities in non-identical dots mentioned above is only due to neglecting higher-order terms in their derivation. We will show that higher-order terms are of fundamental importance. Retaining additional terms in our calculations leads to physically valid results consistent with those of Li *et al* [8]. Our analysis also illustrates the detailed mechanism of quantum measurement.

2. Bloch-type rate equations

The system we study is shown schematically in figure 1. It can be modelled by the Hamiltonian $H = H_{\rm CQD} + H_{\rm QPC} + H_{\rm int}$, with

$$\begin{aligned} H_{\rm CQD} &= E_1 a_1^\dagger a_1 + E_2 a_2^\dagger a_2 + \Omega_0 (a_1^\dagger a_2 + a_2^\dagger a_1), \\ H_{\rm QPC} &= \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} \Omega_{lr} (a_l^\dagger a_r + {\rm H.c.}), \\ H_{\rm int} &= \sum_{l,r} \delta\Omega_{lr} a_1^\dagger a_1 (a_l^\dagger a_r + a_r^\dagger a_l). \end{aligned} \tag{1}$$

Here, $H_{\rm CQD}$ is the Hamiltonian of the CQDs. The Hubbard terms are neglected because we consider only one extra electron in the CQDs. The parameter Ω_0 denotes the hopping amplitude of the extra electron between the two single-dot states (see figure 1). $H_{\rm QPC}$ is the Hamiltonian of the QPC, and $H_{\rm int}$ is the interaction Hamiltonian between the CQDs and the QPC. Electrostatic effects are included in $H_{\rm int}$: an extra electron in the left dot will lead to an effective variation $\delta\Omega_{lr}$ in the coupling Ω_{lr} between the states E_l and E_r in the two reservoirs of the QPC. The resulting coupling then becomes $\Omega'_{lr} = \Omega_{lr} + \delta\Omega_{lr}$.

In the occupation representation, the many-body wavefunction of the entire system (the CQDs plus the QPC) can be written as

$$|\psi(t)\rangle = \Bigg[b_1(t)a_1^\dagger + b_2(t)a_2^\dagger + \sum_{l,r} b_{1lr}(t)a_1^\dagger a_r^\dagger a_l + \sum_{l,r} b_{1rl}(t)a_1^\dagger a_l^\dagger a_r + \sum_{l,r} b_{2lr}(t)a_2^\dagger a_r^\dagger a_l + \sum_{l,r} b_{2rl}(t)a_2^\dagger a_l^\dagger a_r + \sum_{l<l',r<r'} b_{1ll'rr'}(t)a_1^\dagger a_r^\dagger a_{r'}^\dagger a_l a_{l'} + \sum_{l<l',r<r'} b_{1rr'll'}(t)a_1^\dagger a_l^\dagger a_{l'}^\dagger a_r a_{r'} + \sum_{l<l',r<r'} b_{2ll'rr'}(t)a_2^\dagger a_r^\dagger a_{r'}^\dagger a_l a_{l'} + \sum_{l<l',r<r'} b_{2rr'll'}(t)a_2^\dagger a_l^\dagger a_{l'}^\dagger a_r a_{r'} + \cdots \Bigg] |0\rangle \,, \tag{2}$$

where $b_j(t)$, $j = 1, 2, 1lr, 2lr, \ldots$, are the time-dependent probability amplitudes of finding the system in the corresponding states. The vacuum state $|0\rangle$ corresponds to the state of the entire system with no extra electron in the CQDs and before the transfer of any electron between the reservoirs in the QPC. Then, $b_{1lr}(t)$ for instance denotes the probability amplitude that the extra electron is in the left dot and an electron has passed through the barrier in the QPC. The initial conditions are $b_1(0) = 1$ and $b_j(0) = 0$ for $j = 2, 1lr, 2lr, \ldots$. Substituting the many-body wavefunction (equation (2)) and the Hamiltonian (equation (1)) into the time-dependent Schrödinger equation $i|\dot{\psi}(t)\rangle = H|\psi(t)\rangle$ gives an infinite set of linear equations for the probability amplitudes $b_j(t)$. Performing a Laplace transform to each equation, the transformed amplitude $b_j(E) = \int_0^{+\infty} \mathrm{e}^{\mathrm{i}Et} b_j(t)\,\mathrm{d}t$ follows:

$$(E - E_1)b_1(E) - \Omega_0 b_2(E) - \sum_{l,r} \Omega'_{lr} b_{1lr}(E) - \sum_{l',r'} \Omega'_{l'r'} b_{1r'l'}(E) = i, \tag{3a}$$

$$(E - E_2)b_2(E) - \Omega_0 b_1(E) - \sum_{l,r} \Omega_{lr} b_{2lr}(E) - \sum_{l',r'} \Omega_{l'r'} b_{2r'l'}(E) = 0, \tag{3b}$$

$$(E + E_l - E_1 - E_r)b_{1lr}(E) - \Omega'_{lr} b_1(E) - \Omega_0 b_{2lr}(E) - \sum_{l'r'} \Omega'_{l'r'} b_{1ll'rr'}(E) = 0, \tag{3c}$$

$$(E + E_l - E_2 - E_r)b_{2lr}(E) - \Omega_{lr} b_2(E) - \Omega_0 b_{1lr}(E) - \sum_{l'r'} \Omega_{l'r'} b_{2ll'rr'}(E) = 0, \tag{3d}$$

$$(E + E_r - E_1 - E_l)b_{1rl}(E) - \Omega'_{lr} b_1(E) - \Omega_0 b_{2rl}(E) - \sum_{l'r'} \Omega'_{l'r'} b_{1rr'll'}(E) = 0, \tag{3e}$$

$$(E + E_r - E_2 - E_l)b_{2rl}(E) - \Omega_{lr} b_2(E) - \Omega_0 b_{1rl}(E) - \sum_{l'r'} \Omega_{l'r'} b_{2rr'll'}(E) = 0, \tag{3f}$$

$$\cdots\cdots\cdots.$$

To solve equations (3), Gurvitz *et al* [5, 6] introduced an approximation by keeping terms up to the order of $\mathrm{O}(\Omega^2)$ to simplify the corresponding sums (see equations (A5) in [5]). After tracing out the degrees of freedom of the QPC, a coupled set of Bloch-type rate equations are derived. As shown in figure 3 of [5b], however, the resulting occupation probabilities for states $|E_1\rangle$ and $|E_2\rangle$ are 0.5 when $t \to \infty$ for both symmetric and asymmetric CQDs. This is correct for the symmetric case with identical quantum dots, but is unphysical in the latter case. Here, we show that this unphysical feature can be rectified by taking terms of the order of $\Omega^2\Omega_0/V_\mathrm{d}$ or even higher into account. Also, following [5], we consider the high-voltage regime with $eV_\mathrm{d} \gg \Omega^2\rho$, where $V_\mathrm{d} = \mu_\mathrm{L} - \mu_\mathrm{R}$ is the applied gate voltage, and ρ is the density of states in the reservoirs. In this high-voltage regime, when the temperature is low enough (here it is chosen as zero for simplicity, as in [5]), the electron has an extremely low probability to pass through the QPC from the right reservoir (with a lower μ_R) to the left one (with a higher μ_L). Thus, the terms describing the back processes, e.g., $\sum_{l<l',r<r'} b_{1rr'll'}(t)$, can be neglected.

Performing the summation $\sum_{l,r}\Omega'_{lr}b_{1lr}(E)$ in equation (3*a*) for example, we obtain that

$$\sum_{l,r}\Omega'_{lr}b_{1lr}(E)\approx-\frac{\mathrm{i}D'}{2V_\mathrm{d}}(V_\mathrm{d}+E-E_1)b_1(E)+\frac{\mathrm{i}\Lambda}{2V_\mathrm{d}}b_2(E), \tag{4}$$

where $D'=2\pi\rho_l\rho_r\Omega'^2V_\mathrm{d}$, $\Lambda=2\pi\rho_l\rho_r\Omega_0\Omega\Omega'V_\mathrm{d}$, and the corresponding calculations are shown in the appendix. With the same approximate treatment, the resulting equations for $b_j(E)$ are

$$\left[E-E_1+\frac{\mathrm{i}D'}{2V_\mathrm{d}}(V_\mathrm{d}+E-E_1)\right]b_1(E)-\left(\Omega_0+\frac{\mathrm{i}\Lambda}{2V_\mathrm{d}}\right)b_2(E)=\mathrm{i}, \tag{5a}$$

$$\left[E-E_2+\frac{\mathrm{i}D}{2V_\mathrm{d}}(V_\mathrm{d}+E-E_2)\right]b_2(E)-\left(\Omega_0+\frac{\mathrm{i}\Lambda}{2V_\mathrm{d}}\right)b_1(E)=0, \tag{5b}$$

$$\left[E+E_{lr1}+\frac{\mathrm{i}D'}{2V_\mathrm{d}}(V_\mathrm{d}+E+E_{lr1})\right]b_{1lr}(E)-\Omega'b_1(E)-\left(\Omega_0+\frac{\mathrm{i}\Lambda}{2V_\mathrm{d}}\right)b_{2lr}(E)=0, \tag{5c}$$

$$\left[E+E_{lr2}+\frac{\mathrm{i}D}{2V_\mathrm{d}}(V_\mathrm{d}+E+E_{lr2})\right]b_{2lr}(E)-\Omega b_2(E)-\left(\Omega_0+\frac{\mathrm{i}\Lambda}{2V_\mathrm{d}}\right)b_{1lr}(E)=0, \tag{5d}$$

$$\cdots\cdots\cdots,$$

where $D=2\pi\rho_l\rho_r\Omega^2V_\mathrm{d}$ and $E_{lrm}=E_l-E_m-E_r$ $(m=1,2)$. Now, we introduce the notation

$$\sigma_{ij}^{(n)}=\sum_{l\cdots r\cdots}b_{il\cdots r\cdots}(t)b^*_{jl\cdots r\cdots}(t).$$

For instance, $\sigma_{11}^{(1)}=\sum_{lr}b_{1lr}(t)b^*_{1lr}(t)$ denotes the occupation probability for the extra electron staying in the left quantum dot and an electron having passed through the QPC.

The equations for the amplitudes $b_j(E)$, $j=1,2,1lr,2lr,\ldots$, can be converted to a new set of equations for $\sigma_{ij}^{(n)}$ using the inverse Laplace transform:

$$\sigma_{ij}^{(n)}=\sum_{l\cdots r\cdots}\int\frac{\mathrm{d}E\,\mathrm{d}E'}{4\pi^2}b_{il\cdots r\cdots}(E)b^*_{jl\cdots r\cdots}(E')\mathrm{e}^{\mathrm{i}(E'-E)t}.$$

We now multiply equation (5*c*) by $b^*_{1lr}(E')$. The resulting equation is then subtracted by its complex conjugate after exchanging E with E'. We obtain

$$\begin{aligned}\int\int\frac{\mathrm{d}E\,\mathrm{d}E'}{4\pi^2}\sum_{lr}\{&(E'-E-\mathrm{i}D')b_{1lr}(E)b^*_{1lr}(E')\\&-\frac{\mathrm{i}D'}{2V_\mathrm{d}}[(E'+E_{lr1})+(E+E_{lr1})]b_{1lr}(E)b^*_{1lr}(E')\\&+\frac{\mathrm{i}\Lambda}{2V_\mathrm{d}}[b_{2lr}(E)b^*_{1lr}(E')+b_{1lr}(E)b^*_{2lr}(E')]-\Omega_0[b_{1lr}(E)b^*_{2lr}(E')\\&-b_{2lr}(E)b^*_{1lr}(E')]-\Omega'[b^*_1(E')b_{1lr}(E)-b_1(E)b^*_{1lr}(E')]\}\mathrm{e}^{\mathrm{i}(E'-E)t}=0.\end{aligned} \tag{6}$$

Using equations (4), (5*a*) and (5*c*), we can obtain from equation (6), by neglecting terms of order $\mathrm{O}(\Omega^6/V_\mathrm{d}^2)$ and higher, that

$$\begin{aligned}\dot\sigma_{11}^{(1)}=&-D'\left[\sigma_{11}^{(1)}-\sigma_{11}^{(0)}\right]+\mathrm{i}\Omega_0\left[\sigma_{12}^{(1)}-\sigma_{21}^{(1)}\right]\\&+\frac{\Lambda}{2V_\mathrm{d}}\left(1-\frac{\Omega'}{\Omega}\right)\left[\sigma_{12}^{(1)}+\sigma_{21}^{(1)}-\sigma_{12}^{(0)}-\sigma_{21}^{(0)}\right].\end{aligned} \tag{7}$$

Similarly, equation (5*d*) gives

$$\dot{\sigma}_{22}^{(1)} = -D\left[\sigma_{22}^{(1)} - \sigma_{22}^{(0)}\right] + \mathrm{i}\Omega_0\left[\sigma_{21}^{(1)} - \sigma_{12}^{(1)}\right] + \frac{\Lambda}{2V_\mathrm{d}}\left(1 - \frac{\Omega}{\Omega'}\right)\left[\sigma_{12}^{(1)} + \sigma_{21}^{(1)} - \sigma_{12}^{(0)} - \sigma_{21}^{(0)}\right]. \tag{8}$$

To calculate the off-diagonal element σ_{12}, we subtract the product of equation (5*c*) and $b_{2lr}^*(E')$ by the product of $b_{1lr}(E)$ and the complex conjugate of equation (5*d*) after exchanging $E \leftrightarrow E'$. We obtain

$$\begin{aligned}\dot{\sigma}_{12}^{(1)} = {} & \mathrm{i}\varepsilon\sigma_{12}^{(1)} - \tfrac{1}{2}(D' + D)\sigma_{12}^{(1)} + \sqrt{DD'}\sigma_{12}^{(0)} + \mathrm{i}\Omega_0\left[\sigma_{11}^{(1)} - \sigma_{22}^{(1)}\right] \\ & - \frac{\Lambda}{2V_\mathrm{d}}\left[\frac{\Omega'}{\Omega}\sigma_{11}^{(0)} + \frac{\Omega}{\Omega'}\sigma_{22}^{(0)}\right] + \frac{\Lambda}{2V_\mathrm{d}}\left[\sigma_{11}^{(0)} + \sigma_{22}^{(0)}\right] \\ & - \frac{\Lambda}{2V_\mathrm{d}}\left[\frac{\Omega}{\Omega'}\sigma_{11}^{(1)} + \frac{\Omega'}{\Omega}\sigma_{22}^{(1)}\right] + \frac{\Lambda}{2V_\mathrm{d}}\left[\sigma_{11}^{(1)} + \sigma_{22}^{(1)}\right],\end{aligned} \tag{9}$$

where $\varepsilon = E_2 - E_1$ is the energy-level difference of the two quantum dots. Similar procedures can be used for $\sigma_{ij}^{(n)}$ ($n \geqslant 2$), and the resulting equations are

$$\begin{aligned}\dot{\sigma}_{11}^{(n)} = {} & -D'\left[\sigma_{11}^{(n)} - \sigma_{11}^{(n-1)}\right] + \mathrm{i}\Omega_0\left[\sigma_{12}^{(n)} - \sigma_{21}^{(n)}\right] \\ & + \frac{\Lambda}{2V_\mathrm{d}}\left(1 - \frac{\Omega'}{\Omega}\right)\left[\sigma_{12}^{(n)} + \sigma_{21}^{(n)} - \sigma_{12}^{(n-1)} - \sigma_{21}^{(n-1)}\right], \\ \dot{\sigma}_{22}^{(n)} = {} & -D\left[\sigma_{22}^{(n)} - \sigma_{22}^{(n-1)}\right] + \mathrm{i}\Omega_0\left[\sigma_{21}^{(n)} - \sigma_{12}^{(n)}\right] \\ & + \frac{\Lambda}{2V_\mathrm{d}}\left(1 - \frac{\Omega}{\Omega'}\right)\left[\sigma_{12}^{(n)} + \sigma_{21}^{(n)} - \sigma_{12}^{(n-1)} - \sigma_{21}^{(n-1)}\right], \\ \dot{\sigma}_{12}^{(n)} = {} & \mathrm{i}\varepsilon\sigma_{12}^{(n)} - \frac{1}{2}(D' + D)\sigma_{12}^{(n)} + \sqrt{DD'}\sigma_{12}^{(n-1)} + \mathrm{i}\Omega_0\left[\sigma_{11}^{(n)} - \sigma_{22}^{(n)}\right] + \frac{\Lambda}{2V_\mathrm{d}}\left[\sigma_{11}^{(n)} + \sigma_{22}^{(n)}\right] \\ & - \frac{\Lambda}{2V_\mathrm{d}}\left[\frac{\Omega'}{\Omega}\sigma_{11}^{(n-1)} + \frac{\Omega}{\Omega'}\sigma_{22}^{(n-1)}\right] + \frac{\Lambda}{2V_\mathrm{d}}\left[\sigma_{11}^{(n-1)} + \sigma_{22}^{(n-1)}\right] \\ & - \frac{\Lambda}{2V_\mathrm{d}}\left[\frac{\Omega}{\Omega'}\sigma_{11}^{(n)} + \frac{\Omega'}{\Omega}\sigma_{22}^{(n)}\right].\end{aligned} \tag{10}$$

Summing over n, we obtain the following Bloch-type rate equations for the density-matrix elements:

$$\begin{aligned}\dot{\sigma}_{11} &= \mathrm{i}\Omega_0(\sigma_{12} - \sigma_{21}), \\ \dot{\sigma}_{22} &= \mathrm{i}\Omega_0(\sigma_{21} - \sigma_{12}), \\ \dot{\sigma}_{12} &= \mathrm{i}\varepsilon\sigma_{12} - \frac{\Gamma_\mathrm{d}}{2}\sigma_{12} + \mathrm{i}\Omega_0(\sigma_{11} - \sigma_{22}) - \frac{\chi}{2}(\sigma_{11} + \sigma_{22}),\end{aligned} \tag{11}$$

with

$$\Gamma_\mathrm{d} = \left(\sqrt{D'} - \sqrt{D}\right)^2, \qquad \chi = \left(\frac{\Lambda}{V_\mathrm{d}}\right)\left(\frac{\Omega}{\Omega'} + \frac{\Omega'}{\Omega} - 2\right). \tag{12}$$

Here $\sigma_{ij} = \sum_n \sigma_{ij}^{(n)}$ are the reduced density-matrix elements of the CQD system after tracing out the variables of the QPC. The decoherence rate Γ_d characterizes the exponential damping of the off-diagonal density-matrix element. These Bloch-type rate equations are the same as those from a theory based on master equations with the Markovian approximation (cf equation (17) in [8]). When ignoring the effects of the higher-order terms $\chi = 0$. These equations are reduced to the rate equations derived in [5b], which gives the unphysical result of identical occupation

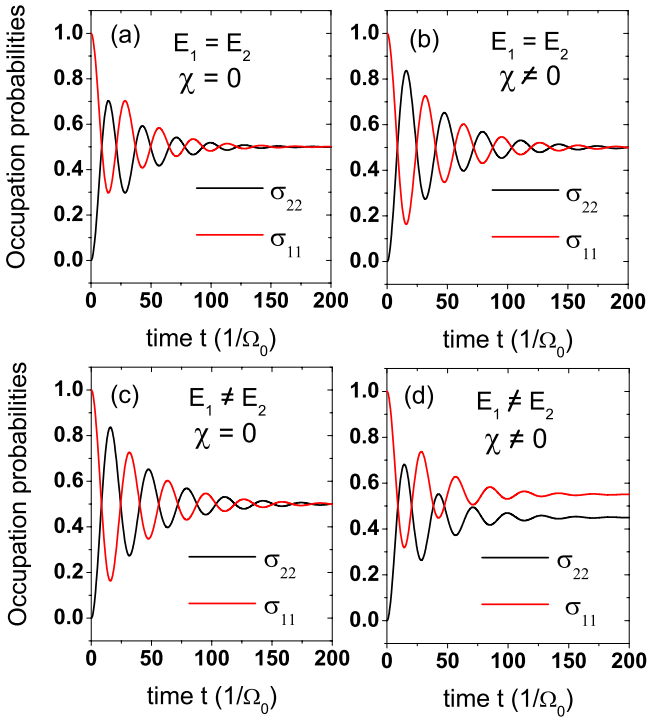

Figure 2. Occupation probability of the extra electron in each quantum dot as a function of time for $\Gamma_d = \Omega_0$ and (a) $\varepsilon = E_2 - E_1 = 0$, $\chi = 0$; (b) $\varepsilon = 0$, $\chi = 0.1\Omega_0$; (c) $\varepsilon = \Omega_0$, $\chi = 0$; and (d) $\varepsilon = \Omega_0$, $\chi = 0.1\Omega_0$.

(This figure is in colour only in the electronic version)

probabilities for asymmetrical CQDs. This implies that the higher-order terms play an essential role in obtaining more accurate and reasonable results and help us better understand the effects of the quantum measurement on the system considered.

Here we take into account the terms of the order $\Omega^2\Omega_0/V_d$, while these terms were ignored in [5]. To obtain more accurate results, one can consider further higher-order terms, which have the order of Ω^6/V_d^2 and $\Omega^6\Omega_0/V_d^3$. In the high-voltage regime, these higher-order terms do not affect the results significantly (see the appendix).

3. Discussion and conclusion

Figure 2 shows the time dependence of the occupation probability of the extra electron in each quantum dot. When $\chi = 0$, the occupation probability in each dot always decays to 0.5 at $t \to \infty$ (see figures 2(a) and (c)). These results are identical to those in [5b]. However, after including higher-order terms, χ becomes nonzero and is not negligible. This gives rise to an additional term $\frac{1}{2}\chi(\sigma_{11}+\sigma_{22})$ in the last equation in (11). From figure 2(b), one can see that the occupation probabilities still decay to 0.5 as $t \to \infty$ for symmetrical CQDs, but decay more slowly than in the previous case with vanishing χ. Moreover, for asymmetrical CQDs, the occupation probability in each dot decays to a different value at $t \to \infty$ (see figure 2(d)). This is reasonable because the two quantum dots are characterized by different parameters. Below we further explain these phenomena analytically.

Let us look at the stationary solution of equation (11) in the limit $t \to \infty$ so that $\dot{\sigma}_{ij}(t \to \infty) = 0$. Using $\sigma_{ij}(t \to \infty) = u_{ij} + \mathrm{i}v_{ij}$, where u_{ij} and v_{ij} are real numbers, equation (11) can be rewritten as

$$\begin{aligned} 0 &= -2\Omega_0 v_{12} \\ 0 &= 2\Omega_0 v_{12} \\ 0 &= \mathrm{i}\varepsilon(u_{12} + \mathrm{i}v_{12}) - \frac{\Gamma_\mathrm{d}}{2}(u_{12} + \mathrm{i}v_{12}) + \mathrm{i}\Omega_0(\sigma_{11} - \sigma_{22}) - \frac{\chi}{2}. \end{aligned} \tag{13}$$

From these equations, one obtains

$$\sigma_{11} - \sigma_{22} = \frac{\chi\varepsilon}{\Omega_0\Gamma_\mathrm{d}} = \frac{\varepsilon}{V_\mathrm{d}}. \tag{14}$$

As expected, $\sigma_{11} = \sigma_{22}$ for $E_1 = E_2$ (i.e., $\varepsilon = 0$), while $\sigma_{11} \neq \sigma_{22}$ for asymmetrical CQDs with $E_1 \neq E_2$ (i.e., $\varepsilon \neq 0$).

In conclusion, we have presented a quantitative description of the dissipative dynamics of a CQD qubit connected to a detector in the form of a QPC. Bloch-type rate equations for the reduced density-matrix elements of the qubit are derived. In particular, the long-time probability distribution of the state of the qubit is found to depend on the energy levels of the single-dot electron states. This corrects an earlier calculation which has predicted a distribution independent of the dot properties. The improvement results from taking into account higher-order interaction terms in our analysis and it can be extended to other quantum systems. In the present work, the Bloch-type rate equations are derived for a CQD system working at zero temperature and applied in the strong bias voltage regime. However, a finite temperature will affect the asymptotic population of the qubit and a population inversion can even be achieved in some cases [9].

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Appendix. Derivation of equations for probability amplitudes

For the QPC, we assume that the hopping amplitude $\Omega_{lr}(E_l, E_r)$ between the left and right reservoirs depends only weakly on the energy levels E_l and E_r. The energy dependence is therefore neglected, i.e., $\Omega_{lr}(E_l, E_r) = \Omega$, and $\Omega'_{lr}(E_l, E_r) = \Omega'$. As high-order terms $\sum_{l'r'} \Omega' b_{1ll'rr'}(E)$ and $\sum_{l'r'} \Omega b_{2ll'rr'}(E)$ are neglected, equations (3*c*) and (3*d*) give

$$\Omega' b_{1lr}(E) \approx \frac{(E + E_l - E_2 - E_r)\Omega'^2 b_1(E) + \Omega'\Omega_0\Omega b_2(E)}{(E + E_l - E_1 - E_r)(E + E_l - E_2 - E_r) - \Omega_0^2}. \tag{A.1}$$

Because the energy levels in each electron reservoir of the QPC are dense, we can replace each sum over l and r in equations (3*a*) and (3*b*) by an integral. For instance, $\sum_{lr} \to \int\int \rho_l(E_l)\rho_r(E_r)\,\mathrm{d}E_l\,\mathrm{d}E_r$, where $\rho_{l,r}$ is the density of states in the left and right reservoirs, respectively. This integral can be split into two parts: the principal and singular value parts. Here the bandwidths of the QPC reservoirs are much larger than V_d and the principal part is thus negligibly small [10]. Actually, the principal part merely renormalizes the energy levels and the singular value part plays the dominant role. When equation (A.1) is substituted into

$\sum_{lr}\Omega' b_{1lr}(E)$, we thus obtain two terms. The first term is given by

$$\begin{aligned}
&\sum_{lr}\frac{(E+E_l-E_2-E_r)\Omega'^2 b_1(E)}{(E+E_l-E_1-E_r)(E+E_l-E_2-E_r)-\Omega_0^2}\\
&\quad= b_1(E)\int_{-\infty}^{\mu_{\rm L}}{\rm d}E_l\int_{\mu_{\rm R}}^{+\infty}{\rm d}E_r\frac{\rho_l(E_l)\rho_r(E_r)\Omega'^2(E+E_l-E_2-E_r)}{(E+E_l-E_r-E_1')(E+E_l-E_r-E_2')}\\
&\quad\approx b_1(E)\int_{-\infty}^{\mu_{\rm L}}{\rm d}E_l\int_{\mu_{\rm R}}^{+\infty}{\rm d}E_r\left(-{\rm i}\pi\rho_l\rho_r\Omega'^2\right)\\
&\quad\times\left\{\frac{E+E_l-E_r-E_2'}{(E+E_l-E_r-E_2')^2+\eta^2}\delta(E+E_l-E_r-E_1')(E+E_l-E_2-E_r)\right.\\
&\quad\left.+\frac{E+E_l-E_r-E_1'}{(E+E_l-E_r-E_1')^2+\eta^2}\delta(E+E_l-E_r-E_2')(E+E_l-E_2-E_r)\right\}\\
&\quad= -{\rm i}\frac{D'}{2V_{\rm d}}b_1(E)\left\{\frac{1}{\varepsilon'}\left(\frac{\varepsilon'}{2}+\frac{\varepsilon}{2}\right)(V_{\rm d}+E-E_1')\theta(V_{\rm d}+E-E_1')\right.\\
&\quad\left.+\frac{1}{\varepsilon'}\left(\frac{\varepsilon'}{2}-\frac{\varepsilon}{2}\right)(V_{\rm d}+E-E_2')\theta(V_{\rm d}+E-E_2')\right\},
\end{aligned}\tag{A.2}$$

where $V_{\rm d}=\mu_{\rm L}-\mu_{\rm R}$, $D'=2\pi\rho_l\rho_r\Omega'^2V_{\rm d}$, $\varepsilon=E_2-E_1$, $\varepsilon'=\sqrt{\varepsilon^2+4\Omega_0^2}$, $E_{1,2}'=\frac{1}{2}(E_1+E_2)\mp\frac{1}{2}\varepsilon'$, and $\theta(x)$ is the Heaviside step function. In the high-voltage limit with $eV_{\rm d}\gg\Omega^2\rho$, the step function in equation (A.2) becomes one. Thus, the left-hand side of equation (A.2) can be approximated by $-{\rm i}(D'/2V_{\rm d})(V_{\rm d}+E-E_1)b_1(E)$. When higher-order terms, i.e., the terms of the order $\mathrm{O}(\Omega^6/V_{\rm d}^2)$, are included, equation (A.2) is approximated by $-{\rm i}(D'/2V_{\rm d})\{V_{\rm d}[1-\frac{1}{2}(D'/V_{\rm d})^2]+E-E_1\}b_1(E)$ in the high-voltage limit. Obviously, the correction by $(D'/V_{\rm d})^2$ is very small for a large voltage $V_{\rm d}$.

The second term is

$$\begin{aligned}
&\sum_{lr}\frac{\Omega'\Omega\Omega_0 b_2(E)}{(E+E_l-E_1-E_r)(E+E_l-E_2-E_r)-\Omega_0^2}\\
&\quad= b_2(E)\int_{-\infty}^{\mu_{\rm L}}{\rm d}E_l\int_{\mu_{\rm R}}^{+\infty}{\rm d}E_r\frac{\rho_l(E_l)\rho_r(E_r)\Omega'\Omega\Omega_0}{(E+E_l-E_1'-E_r)(E+E_l-E_2'-E_r)}\\
&\quad\approx b_2(E)\int_{-\infty}^{\mu_{\rm L}}{\rm d}E_l\int_{\mu_{\rm R}}^{+\infty}{\rm d}E_r(-{\rm i}\pi\rho_l\rho_r\Omega'\Omega\Omega_0)\\
&\quad\times\left\{\frac{1}{E+E_l-E_1'-E_r}\delta(E+E_l-E_2'-E_r)\right.\\
&\quad\left.+\frac{1}{E+E_l-E_2'-E_r}\delta(E+E_l-E_1'-E_r)\right\}\\
&\quad= \frac{-{\rm i}\Lambda}{2V_{\rm d}}b_2(E)\frac{1}{E_2'-E_1'}[(V_{\rm d}+E-E_2')\theta(V_{\rm d}+E-E_2')\\
&\quad-(V_{\rm d}+E-E_1')\theta(V_{\rm d}+E-E_1')],
\end{aligned}\tag{A.3}$$

where $\Lambda=2\pi\rho_l\rho_r\Omega'\Omega_0\Omega V_{\rm d}$. In the high-voltage limit, the two step functions in equation (A.3) become one and the left-hand side of equation (A.3) is finally approximated by ${\rm i}(\Lambda/2V_{\rm d})b_2(E)$. Thus, the sum of these two terms gives equation (4) in the high-voltage limit. Substituting equation (4) into equation (3*a*), one obtains equation (5*a*). Also, one can derive equations (5*b*)–(5*d*) using similar procedures. When higher-order terms, i.e., the terms of the order $\mathrm{O}(\Omega^6\Omega_0/V_{\rm d}^3)$, are considered, equation (A.3) is approximated by ${\rm i}(\Lambda/2V_{\rm d})(1+\gamma)b_2(E)$

in the high-voltage regime, where the correction $\gamma = [(D + D')\sqrt{DD'} - DD']/4V_d^2$ is very small for a large voltage V_d.

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